NOTATION

Here p, r, t, are pressure, coordinate, time; h, τ, coordinate and time steps; u, v, grid functions; q, well yield; r_0, radius of well; a, ε, g, system parameters; a superimposed circumflex denotes values of the grid functions for the next time layer.

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DESCRIPTION OF THE STRENGTH OF POROUS BODIES ON THE BASIS OF PERCOLATION THEORY

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A method is proposed for estimating the strength of porous materials in destructive rupture on the basis of the percolation theory of regularly packed spheres. The results of calculation by this method are in good agreement with experimental data in the whole porosity range of the material.

Percolation theory [1, 2] is widely used to describe various probabilistic processes. There have been numerous studies of the elasticity of two- and three-dimensional percolation systems, for example [3-7]; it was noted in [4, 5] that the elasticity and conductivity problems for the volume elastic modulus of gel at the gel point belong to different universality classes. In [7], the method of reduction to an elementary cell was used to determine the moduli of the percolation systems, permitting considerable simplification of the calculations. An analogy between the mechanical characteristics and thermal conductivity of porous powder materials was made in [8]. In [9], percolation theory was used to describe the strength and rheological characteristics of disperse systems; however, the use of Bethe lattices limits the application of the given approach to transitions of sol-gel type. The percolational approach to the description of the strength of porous media was considered in [10]; the dependence of the relative strength G (short-term resistance of the material in rupture referred to its maximum value) on the relative particle concentration $m/m_0 = (1 - \Pi)/(1 - \Pi_0)$ where Π_0 is the porosity of the system corresponding to the densest random packing of the particles, was derived. One advantage of this approach is the elimination of the traditional consideration of the strength using the concept of the "failure surface," which leads to complication in taking account of statistical inhomogeneities of the system, for example, macropores. The calculation method is based on an analogy between conduction and the strength of a percolation system, but this analogy is more explicit if the proportion of conducting point of the percolational system is identified with the mean relative coordination number $\nu = (Z + 1)/(Z_0 + 1)$, which characterizes the proportion of possible particle bonds realized on average in the given case.

An analytical expression from [11] is used to calculate the mean coordination number of a lattice of randomly packed spherical particles as a function of the porosity of the packing

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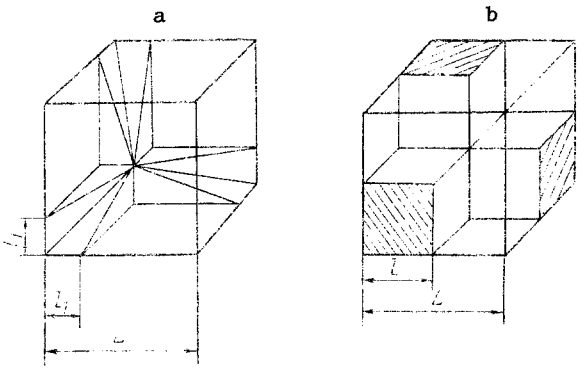

Fig. 1

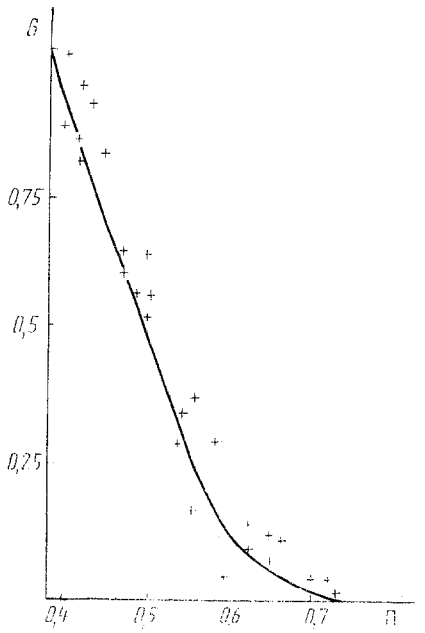

Fig. 2

Fig. 1. Generalized form of inhomogeneous model: a) general view; b) model with interpenetrating components.

Fig. 2. Dependence of the relative strength of a body on its porosity: points) experimental data; curve) calculation by the method here proposed.

$$Z = \frac{\Pi + 3 + \sqrt{\Pi^2 - 10\Pi + 9}}{2\Pi}. \tag{1}$$

The coordination Z_0 of a lattice of randomly close-packed spheres (the porosity Π_0 is 0.39 according to [10]) is 7.28 according to Eq. (1).

To determine the percolation threshold of the given lattice, it is convenient to use the dependence

$$\nu = 0{,}15/(1 - \Pi_c), \tag{2}$$

which provides a good description of the influence of porosity on the percolation threshold of regular three-dimensional lattices [2].

It follows from Eqs. (1) and (2) that

$$\frac{(\Pi_c + 3 + \sqrt{\Pi_c^2 - 10\Pi_c + 9})/(2\Pi_c) + 1}{(\Pi_0 + 3 + \sqrt{\Pi_0^2 - 10\Pi_0 + 9})/(2\Pi_0) + 1} = \frac{0{,}15}{1 - \Pi_c}. \tag{3}$$

The threshold lattice porosity $\Pi_c = 0.729$ is obtained from Eq. (3) by an iterative method for $\Pi_0 = 0.39$. Then, substituting this value of Π_c into Eq. (1), the coordination number of the lattice $Z_c = 3.584$ is determined and hence the percolation threshold $\nu_c = 0.553$.

Knowing ν_c, the relative strength close to the percolation threshold may be calculated using the scaling asymptote.

Far from the percolation threshold, the dependence of the lattice conductivity on its coordination number is used, taking account of the analogy between the strength and conductivity of percolational systems [10]

$$G(\nu) = 1 - \frac{2Z - 2}{Z - 2}(1 - \nu). \tag{4}$$

Substituting Eq. (1) into Eq. (4), it is found that

$$G(\Pi) = 1 - \frac{(\Pi + 3 + \sqrt{\Pi^2 - 10\Pi + 9})/\Pi - 2}{(\Pi + 3 + \sqrt{\Pi^2 - 10\Pi + 9})/(2\Pi) - 2}\left(1 - \frac{(\Pi + 3 + \sqrt{\Pi^2 - 10\Pi + 9})/(2\Pi) + 1}{8{,}2843}\right). \quad (5)$$

Analogous expressions for the relative strength and scaling asymptote obtained in [10] are not consistent, which prevents the determination of intermediate values of the relative strength and considerably reduces the value of the given approach. To determine intermediate values of the strength, the following procedure is adopted.

Equation (5) is valid as $\nu \to 1$. To determine its limit of applicability in the other direction, the method of reduction to an elementary cell is used [11]. For maximum correspondence with the physical essence of the given problem, the method is modified as follows. The form of the isolated cluster in the elementary cell is changed so that the system becomes bound on reaching a threshold relative concentration m_c, but has zero conductivity (Fig. 1a). This change corresponds to a greater extent with the physical essence of the percolation processes. The maximum core size of the isolated cluster l_c in the given model of a heterogeneous medium is determined from the expression

$$m_c = 2(l_c/L)^2 - (l_c/L)^3. \quad (6)$$

For $\Pi_c = 0.729$, the result $l_c/L = 0.4133$ is obtained.

With further reduction in porosity in this model increase in disperse-phase concentration m corresponds to increase in cross section of the bridges between isolated clusters up to the formation of a model with interpenetrating components (Fig. 1b).

The minimal disperse-phase concentration in the formation of a model with interpenetrating components is written in the form

$$m_{\text{in.min}} = 3(l_c/L)^2 - 2(l_c/L)^3. \quad (7)$$

For $m_c = 0.271$ and $l_c/L = 0.4133$, it is found that $m_{\text{in.min}} = 0.442$, $Z_{\text{in.min}} = 4.919$, $\nu_{\text{in.min}} = 0.7145$, and $\Pi_{\text{in.max}} = 0.558$.

Substituting $\Pi_{\text{in.max}}$ into Eq. (5) gives $G(\Pi_{\text{in.max}}) = 0.233$.

The law of variation in cross section of the bridges between the isolated clusters, taking account of the complex topology of the infinite cluster and the probabilistic character of its formation and also ensuring smooth transition in Eq. (5) at $P_{\text{in.max}}$, may be written in the form

$$G(\Pi) = G(\Pi_{\text{in.max}})\left(\frac{\nu - \nu_c}{\nu_{\text{in.min}} - \nu_c}\right)^t, \quad (8)$$

$$\nu_c \leqslant \nu \leqslant \nu_{\text{in.min}},$$

where $t = 1.8 \pm 0.2$.

For $\Pi_{\text{in.max}} = 0.558$, $\nu_c = 0.553$, $G(\Pi_{\text{in.max}}) = 0.233$, and $t = 1.8$, Eq. (8) takes the form

$$G(\Pi) = 0{,}233\left[\frac{(\Pi + 3\sqrt{\Pi^2 - 10\Pi + 9})(2\Pi) + 1}{1{,}3379} - 3{,}4241\right]^{1.8}, \quad (9)$$

$$0{,}558 \leqslant \Pi \leqslant 0{,}729.$$

Curves of $G(\Pi)$ calculated from Eqs. (3), (5)-(7), and (9) with $\Pi_0 = 0.39$ are shown in Fig. 2 in comparison with experimental data obtained in the extension of samples made from polystyrene powder [10]. It is evident that the theoretical curve and the experimental data are in sufficiently good agreement over the whole range of porosity variation of the system.

CONCLUSION

The method proposed in the present work for describing the strength of disperse systems allows theoretical expressions that are in good agreement with experimental data to be obtained for samples obtained by sintering

randomly packed particles, over the whole range of porosity variation. This method may be used to predict the strength of porous materials.

NOTATION

Here G is the relative strength of porous materials; m, volume concentration of solid phase; Π, porosity of system; Π_0, porosity of system corresponding to the densest random packing; Z, coordination number of system; ν, relative coordination of system; ν_c, percolation threshold; l_c, maximum core size of isolated cluster; L, size of macroscopic cube, no smaller than the correlation radius between conducting particles; Π_c, threshold porosity of system.

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